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Numerical modeling of single-layer electroactive polymer mirrors for space applications

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ABSTRACT

Thin-film mirrors are attractive for large apertures, lightweight optical systems and microwave antennas operating in micro-gravity environments. The surface shape of these deployable thin film structures requires control to a precision range that depends on the specific applications. For optical systems, such surfaces need to be deployed and refined in the range of sub-microns. Electroactive polymers (EAP) are potential candidates for making such thin film materials. Generally, EAP materials are produced in thin film form with electrodes on their major surfaces. Depending on the reflectivity of the electrodes and surface roughness of the polymer they can also be produced with mirror finishes. The development of a controllable mirror made of single-layer EAP mirror was investigated and the results are described in this manuscript. The analytical solution for the required voltage/strain distribution to allow forming a parabolic mirror from a planar film is presented. Calculations show a single layer film made of currently available EAP has the capability to control the focal length of a 2-m diameter mirror from infinity to 1.25 m. The results are verified by FEM simulations.

Keyword: Electroactive polymer, thin film mirror, thin film antenna, reflector, controllable membranes.

1. INTRODUCTION

Thin-film mirrors are attractive for large apertures, lightweight optical systems and microwave antennas operating in micro-gravity space. The surface shape of these deployable thin film structures requires control to a precision range that depends on the specific applications. For optical systems, such surfaces need to be deployed and refined in the range of sub-microns. Numerous mechanisms to control mirrors were proposed [1-4]. Electroactive polymers (EAP) are one of potential candidates of the actuation materials [5].

The EAP materials can be categorized to two main classes. One is the ionic EAP [6-9]. The ionic EAP's contain electrolyte in a polymer frame. They show large bending deformation at low voltage excitation. However, the actuation mechanism of these materials involves transport of ions and molecules from one side to the other of the film. The component and properties of the electrolyte have to be maintained well to keep the performance stable. This type EAP may not be a good candidate for space application due to the volatility of the electrolyte. Another class is electric field EAP [10-12]. It includes piezoelectric, electrostrictive, ferroelectric or dielectric polymers. These materials behave like capacitors under excitation voltages and the strain is usually proportional to the square of the applied voltage. Although they require relative high voltage (10 ~ 100V/μm) to create maximum strain, these polymers are dry and have relatively stable performances. The study presented in this paper focuses on electric field EAP's. The strain of these EAP's can be as high as several to ten percents. An example is a P(VDF-TrFE)-based Co/Ter-polymer having a maximum transverse strain of ~4% [11].

One advantage of using EAP is the EAP film can be both the structural and the actuation material of the mirrors. Generally, EAP's are produced in thin film form with electrodes on their major surfaces. Depending on the reflectivity of the electrodes and surface roughness of the polymer they can also be produced with mirror finishes.

The major advantage of the EAP's is the capability to realize distributed actuation to the whole mirror surface. PVDF bimorphs were suggested for distributed shape control [2]. The required electric charge for the shape control can be deposited wirelessly by using electron guns. In order to control the curvature in two directions on the mirror surface, the authors suggested exciting both sides of the bimorph. This, together with the anisotropy of the PVDF material properties, results in tremendous complexity for the control. The uniformity of the glue layer in the bimorph also is a difficult factor in practice.

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In this paper, we propose a controllable mirror made of single-layer EAP. The EAP can be an isotropic, electric field activated material. An analytical solution of required voltage distribution for forming a parabolic mirror from an initially planar film is derived. Calculations show a single layer film made of currently available EAP has the capability to adjust the focus distance of 2-m mirror from infinity to 1.25 m. The results are verified by FEM model.

2. SINGLE-LAYER CONTROLABLE EAP MEMBRANE MIRROR

The EAP bimorph actuators show impressive bending deformation. Obviously, the bimorph actuators are able to change the curvature and the shape of thin film mirror. However, it is not necessary to use bimorphs that usually require two-layer configuration. Single layer EAP mirrors may change their shapes and curvatures under applied voltage. For an initially curved single-layer EAP mirror, ones also expect some shape change because of the strain introduced by the voltage. For the case of initially flat mirror, the results will be a little complicated. A uniform planar EAP film with no constraints will only change its size but curvature under a uniformly distributed voltage. However a non-uniformly distributed voltage or extra boundary constraints can result into a curved shape. An example is the way woks were made in the early years using a hammer. One hits the area near the center of an initially flat plate more times than the metal in the surrounding area. It makes the center part more extended than the outside creating the curved wok shape. The same principle can be applied to an EAP film mirror. An applied voltage which decreases from center to the edge may deform the mirror from flat to desired optical shape. A detailed theoretic analysis for this principle applied to a circular membrane is described in the following paragraphs.

The desired optical mirror is a paraboloid. We express the curve as

$$z = ar^2, \quad (0 \le r \le r_1) \tag{1}$$

Suppose an original EAP film of the same diameter is deformed to the paraboloid by area extension due to the applied electric field across the thickness as shown in Figure 1.

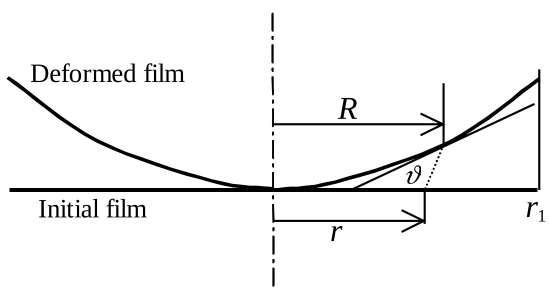

Figure 1. Paraboloid surface deformed from a planar disk.

We also assume the film is thin enough, so the bending stiffness can be neglected, (i.e. we can treat it as a membrane). To avoid buckling of the thin film, any negative in-plan stress (compression) should be avoided in the deformed film. Here we set the in-plan stress to zero. Therefore, the strain in any area of the deformed film is the same as that with free boundary condition. For isotropic film materials, we have

$$S_c = S_r . \tag{2}$$

where the S denotes the strain, subscript *c* denotes the circumferential direction and *r* the tangent direction of the curve.

We define R(r) as the deformed radius of the original circle of radius of r. We have

$$S_c = \frac{R(r)}{r} - 1, \tag{3}$$

and

$$S_r = \frac{R'(r)}{\cos(\vartheta)} - 1, \tag{4}$$

where the ϑ is the angle of tangent of the curve at radius of R. And we have

$$\frac{1}{\cos(\vartheta)} = \sqrt{1+\tan^2(\vartheta)} = \sqrt{1+(z')^2} = \sqrt{1+4a^2R^2}\ . \tag{5}$$

The differential equation is established as

$$R'(r) = \frac{R}{r\sqrt{1+4a^2R^2}}\ . \tag{6}$$

The differential equation has a solution of the form

$$\sqrt{1+4a^2R^2} - a\tanh\left(\frac{1}{\sqrt{1+4a^2R^2}}\right) = \ln(r) + c\ , \tag{7}$$

where the c is a constant that can be determined by boundary condition. If we set the diameter of the deformed mirror equal to the initial diameter, the boundary condition is

$$R(r_1) = r_1, \tag{8}$$

and the constant is

$$c = \sqrt{1+4a^2r_1^{\ 2}} - a\tanh\left(\frac{1}{\sqrt{1+4a^2r_1^{\ 2}}}\right) - \ln(r_1)\ . \tag{9}$$

A computed example is a mirror of 2 m in diameter. The target parabolic curve is $z = 0.2r^2$. The required strain to from such a mirror from a planar film of the same diameter is calculated and shown in Figure 2. As predicted, the required extension of the film has a maximum at the center and decreases with the radius. The maximum value is 4% that is available with the electrostrictive EAP material. The focus distance is

$$l = \frac{1}{4a}\ , \tag{10}$$

and equals to 1.25 m for this example. The f-number of the mirror, which is the ratio of the focus distance over the diameter, is

$$f = \frac{1}{8ar_1}\ . \tag{11}$$

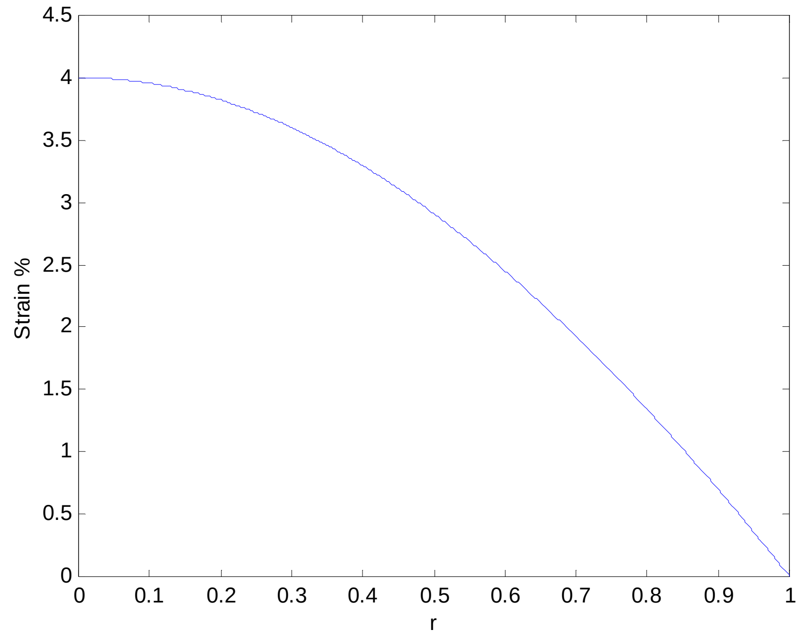

Figure 2. Required extension strain to form the parabolic mirror with $z = 0.2\, r^2$ from a planar sheet.

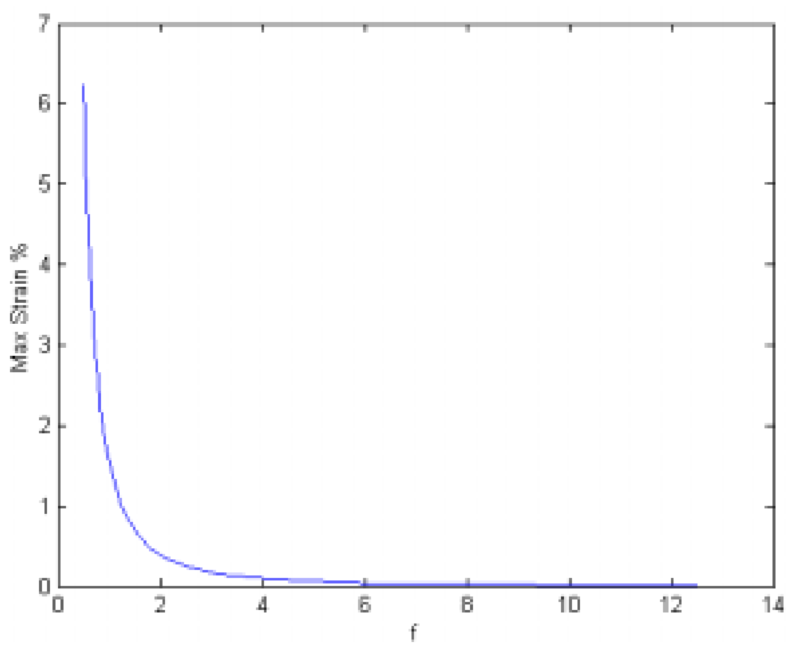

Figure 3. Maximum required extension strain as function of f-number of the parabolic mirror formed from a planar sheet.

This example mirror is operating at f/0.625.

A further investigation of the Eq.(7) and Eq.(9) shows that the required maximum strain is a function of ar_1 i.e. a function of f-number. The function is computed numerically and illustrated in Figure 3.

3. FINITE ELEMENT SIMULATION

A finite element model was developed to verify the analytical results. Two EAP films with different thickness were simulated by FEM model. The geometry and material properties used in the simulation are listed in Table 1. The planar films were expressed by 100 shell elements (Shell-l51, ANSYS). The extension strain created by the electric field in the EAP films was simulated by thermal expansion by set proper material thermal coefficient and corresponding temperature change. "Large Deflection" function of the ANSYS was activated to take the geometry nonlinear effect into account. The shell element does not support a 'buckling' solution. Therefore, the direct solution of this model will be a planar film with in plan strain and stress. Physically, it usually is an unstable state for thin film or membrane because of compression stress in certain areas. Any out-plan perturbation will let the film bend to a stable state, either to one side or the other. To get the stable state solution, an artificial pressure had to be added first and was taken out after the first solution step. The pressure had the effect of producing an initial force on one side of the film.

Table 1. Parameters used in FEM simulation

Film NO.	Diameter (m)	Thickness (μm)	E (Gpa)	Poisson's ratio σ
1	2	100	1	0.3
2	2	10	1	0.3

The results calculated by the FEM model are shown in Figure 4. The curve was fitted to the parabolic formula as

$$z = ar^2 + b\,. \tag{12}$$

No difference between the FEM results and the fitted curve can be found in the scale of the figure 4 for both films No.1 and No.2. We concluded that the analytical formula is correct. However, a closer view of the data showed aberrations. The analytical solution is derived under the membrane assumption. Therefore, the thickness, which introduced non-zero bending stiffness to the shell, may result in aberration from the parabolic curve. The parabolic aberration defined as the difference of the surface from the paraboloid is presented in Figure 5 and 6 for film thickness of 100 μm and 10 μm respectively. Both aberrations are at the level of a few micrometers and show small difference. Because the coordinate shifts 0.2 m in the FEM simulation, the parameters of the target curve are a = 0.2 and b = -0.2. The departures of parameters of the fitted curve from the target parabolic curve, which are in the level of 10^{-6}, are listed in Table2. The difference between the two films is also small. The comparison of the results for the two films implies that the effect of thickness, when it is less than 100 μm for 2 m diameter mirror, is rather small and the aberrations shown in both figures and the errors of the parameters of the fitted curves are mainly due to the accuracy of the numerical model.

Table 2. The departure of the parameters of the fitted curves from the target curve (a=0.2; b=-0.2)

Film No.	Thickness (μm)	Δa	Δb
1	100	4.097×10^{-6}	-1.472×10^{-6}
2	10	4.166×10^{-6}	-1.303×10^{-6}

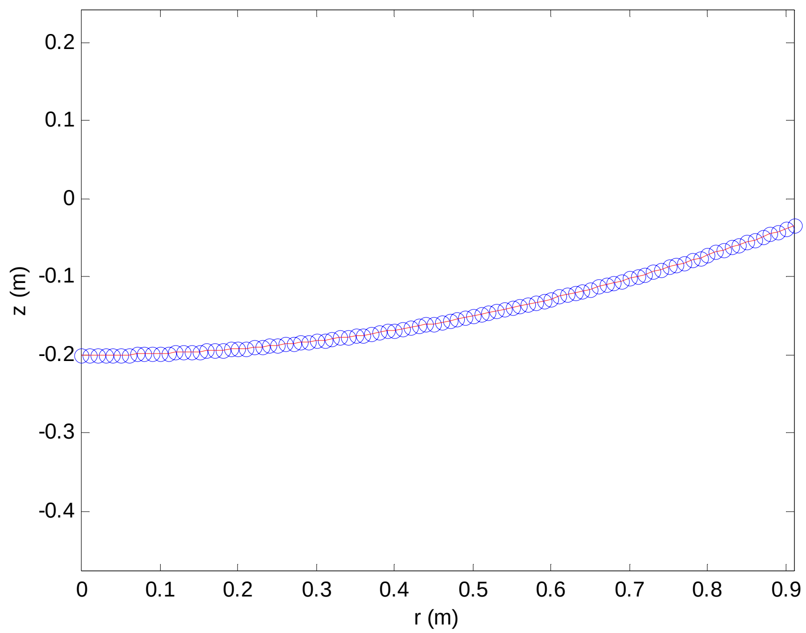

Figure 4. Deformed shape computed by FEM model (circle) and fitted curve of $z = ar^2 + b$ (line).

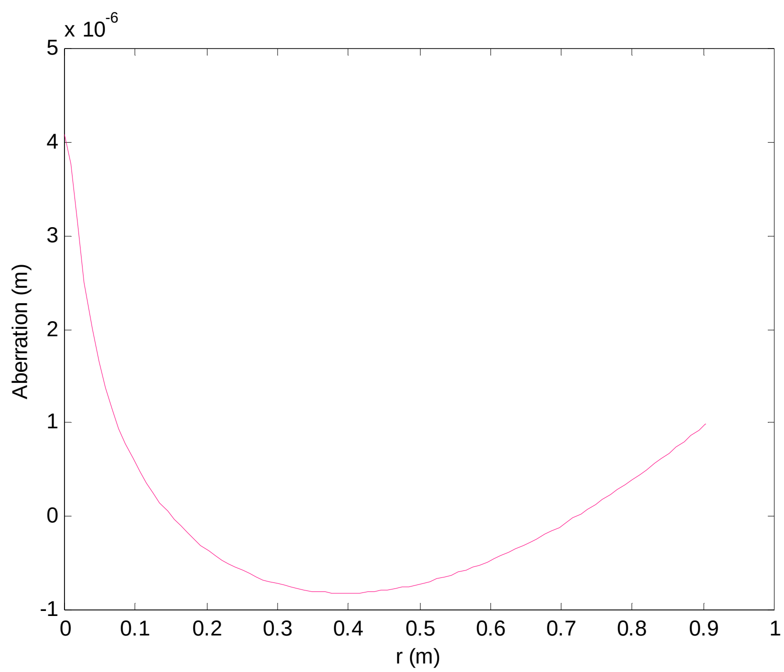

Figure 5. Aberration of the FEM results from the paraboloid, film thickness = 100 μm.

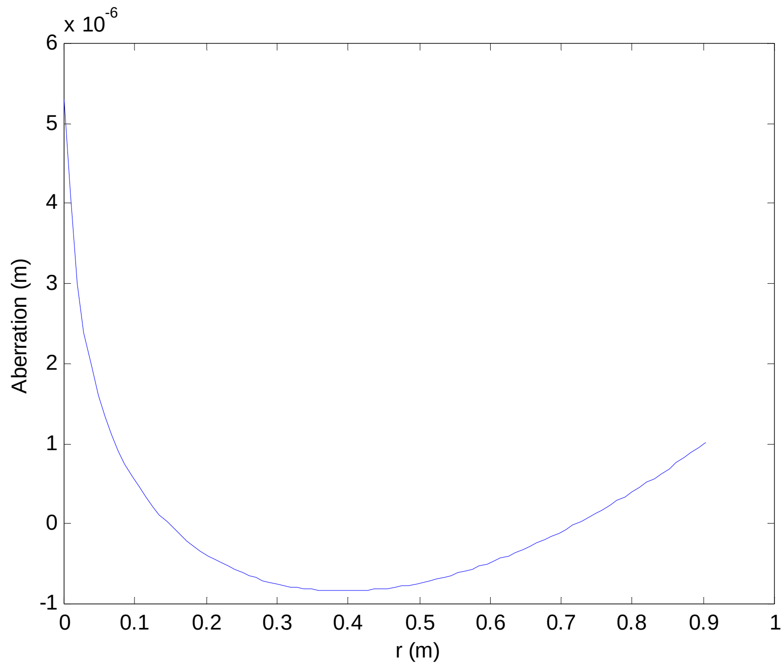

Figure 6. Aberration of the FEM results from the paraboloid, film thickness = 10 μm.

4. CONCLUSIONS

A controllable mirror made of single-layer EAP film was proposed in this paper. Applying a distributed voltage to the backside of the EAP mirror was shown to control the shape of the mirror. The voltage may be applied wirelessly by using electron gun as suggested in literature [2]. This control mechanism could be utilized along or combined with other control methods such as pressure in inflatable structure.

An analytical solution of required voltage/strain distribution for forming a parabolic mirror from a planar membrane was found explicitly. Computed results show a mirror made of single layer EAP film with maximum strain of 4% is able to control the focus distance of a 2-m mirror from infinity to 1.25 m.

The analytical solutions were verified by a FEM model numerically. The FEM simulated results confirmed that the solution was correct. The numerical results also implied that the effect of the thickness of the film was not significant for a 2-m mirror if the thickness of the film is less than 100 μm. An improvement of the accuracy of the FEM model is required to determine the thickness effect in the level of submicrons.

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